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# PROBLEMS AND SOLUTIONS.

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## PROBLEMS FOR SOLUTION.

**2814. Proposed by NATHAN ALTSHILLER, University of Oklahoma.**

The bisectors of the angles formed by the diagonals of an inscribed quadrilateral are: (1) parallel to the lines joining the midpoints of the arcs subtended by the opposite sides of the quadrilateral on its circumcircle; (2) parallel to the bisectors of the angles formed by any pair of opposite sides of the quadrilateral; (3) equally inclined to pairs of sides of the quadrilateral.

**2815. Proposed by the late L. G. WELD.**

A right circular cone is laid upon an inclined plane so that its element of contact makes a given angle with the slant line of the plane. Assuming that there is no slipping and that the rolling friction is negligible, find the time of oscillation of the cone.

**2816. Proposed by W. H. ECHOLS, University of Virginia.**

Find two points $D$ and $E$ on the sides $AB$ and $CB$, respectively, of a triangle $ABC$ such that $AD = DE = EC$.

Give a rule and compass construction.

**2817. Proposed by W. D. CAIRNS, Oberlin College.**

The normal probability curve is sometimes called the binomial curve from the correspondence of its ordinates to the terms of $(1 + 1)^k$. Find an expression for the "mean deviation" of these terms from the median for the case where $k = 2m$. Assume the scale division as unity.

**2818. Proposed by S. A. COREY, Des Moines, Iowa.**

What is the maximum error that could occur in computing the common logarithm of $(1 + x)$ by the formula:

$$\text{Log}_{10}(1 + x) = .144{,}764{,}827\,\frac{x}{1 + x} + .579{,}059{,}309\,\frac{x}{2 + x} + .072{,}151{,}17\,\frac{x^2}{1 + x},$$

where $0 \leqq x \leqq 6/10$?

**2819. Proposed by B. F. FINKEL, Drury College.**

Find the equation of the envelope of the system of circles inscribed in a triangle with a given base and a given vertical angle.

**2820. Proposed by C. B. HALDEMAN, Ross, Ohio.**

Given one angle and the radii of the inscribed and circumscribed circles, to construct the triangle geometrically.

**2821. Proposed by FRANK IRWIN, University of California.**

The quantities $x_1, x_2, \cdots x_n$ vary in such a way that their sum (or any other one of the elementary symmetric functions) remains constant; investigate the maxima and minima of the remaining elementary symmetric functions.

## SOLUTIONS OF PROBLEMS.

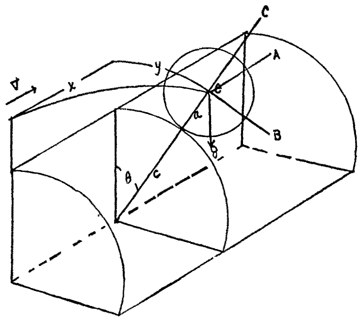
**272 (Mechanics) [1913, 64; 1919, 213]. Proposed by J. F. LAWRENCE, Stillwater, Okla.**

A perfectly rough circular cylinder is fixed with its axis horizontal. A sphere is placed on it in a position of unstable equilibrium, and projected with a given velocity parallel to the axis of

the cylinder. If the sphere be slightly disturbed in a horizontal direction perpendicular to the direction of the axis of the cylinder, determine at what point the sphere will leave the cylinder.

## Solution by J. B. Reynolds, Lehigh University.

Assume as moving axes the following mutually perpendicular lines through the center of the sphere: $GA$ parallel to the axis of the cylinder, $GC$ along the common normal and $GB$. Let the velocity of the sphere parallel to $GA$ be $u$ and to $GB$ be $v$. Then if $\omega_3$ be the angular velocity of the sphere about $GC$ and $\rho$ the radius of curvature of the circular section of the surface upon which the center of the sphere $G$ moves, we have the equations of motion[1]




$$\frac{du}{dt} = \frac{a^2}{a^2+k^2}X - \frac{k^2}{a^2+k^2}\frac{a\omega_3 v}{\rho}, \qquad (1)$$

$$\frac{dv}{dt} = \frac{a^2}{a^2+k^2}Y, \qquad (2)$$

and

$$\frac{a\,d\omega_3}{dt} = \frac{uv}{\rho}, \qquad (3)$$

$X$ and $Y$ being the impressed forces acting on the sphere parallel to $GA$ and $GB$, and $a$ and $k$ the radius and radius of gyration of the sphere, respectively. Assuming the sphere to be of unit mass and the cylinder of radius $c$, we have

$$X = 0 \quad Y = g\sin\theta, \quad (4) \qquad \rho = a + c, \quad (5)$$

$$y = (a+c)\theta, \ \text{(see figure)}, \quad (6) \qquad v = (a+c)\dot\theta, \quad (7) \qquad k^2 = (2/5)a^2.$$

By these, we get from (1), (2), and (3)

$$du/dt = -(2/7)a\omega_3\dot\theta, \quad (8) \qquad (c+a)\ddot\theta = (5/7)g\sin\theta, \quad (9)$$

$$a\,d\omega_3/dt = u\dot\theta. \quad (10)$$

By (8) and (10), $udu + (2/7)(a\omega_3)d(a\omega_3) = 0$. Hence

$$u^2 = -(2/7)(a\omega_3)^2 + V^2, \qquad (11)$$

for $u = V$ when $\omega_3 = 0$, $V$ being the initial velocity of projection. By (10) and (11), $a\,d\omega_3/dt = \sqrt{V^2 - (a\omega_3)^2\frac{2}{7}}\,d\theta/dt$; whence, $\sin^{-1}(\sqrt{2/7}\,a\omega_3/V) = \sqrt{2/7}\,\theta$, for $\omega_3 = 0$ when $\theta = 0$, or

$$\sqrt{2/7}\,a\omega_3 = V\sin\sqrt{2/7}\,\theta. \qquad (12)$$

By (12) and (11), $u^2 = V^2 - V^2\sin^2\sqrt{2/7}\,\theta = V^2\cos^2\sqrt{2/7}\,\theta$. Hence,

$$u = V\cos\sqrt{2/7}\,\theta. \qquad (13)$$

If $R$ is the reaction between the sphere and the cylinder, we have $g\cos\theta - R = (c+a)\dot\theta^2$, and when the sphere leaves the cylinder $R = 0$, or

$$g\cos\theta = (c+a)\theta^2. \qquad (14)$$

By (9), $(c+a)(\dot\theta^2/2) = (5/7)g(1-\cos\theta)$; so that by (14) when they separate,

$$g\cos\theta = (10/7)g(1-\cos\theta),$$

or $\cos\theta = 10/17$. The distance traversed parallel to the axis of the cylinder from the position of slight displacement is

$$x = \int u\,dt = \int \frac{u}{\dot\theta}\,d\theta = \frac{V}{10}\sqrt{35}\sqrt{\frac{c+a}{g}}\int_{\theta_1}^{\cos^{-1}\frac{10}{17}} \cos\sqrt{2/7}\,\theta\,\csc\frac{\theta}{2}\,d\theta,$$

by (12) and (14), $\theta_1$ being the initial angle of slight displacement.

[1] *Cf.* Routh, *Advanced Rigid Dynamics*, sixth edition, 1905, p. 172, article 225.—Editors.